Summary for hydrodynamic interactions (HI) between particles

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1. Mobility tensor

Suppose that a collection of N spherical particles, all having the same radius a, are suspended in an incompressible fluid with the viscosity η. Let $\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_N$ be the positions of the particles and $\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_N$ be the force acting on them. We assume that there are no external torques acting on the particles and inertia effects are all neglected due to very small Re. Then the velocities of the particles are written as

$$\mathbf{V}_n = \sum_{m=1}^{N} \mathbf{H}_{nm} \cdot \mathbf{F}_m$$

by using the mobility tensor $\mathbf{H}_{nm}$. Three representations of $\mathbf{H}_{nm}$ with different levels of approximations are summarized below.

I) No HI:

$$\mathbf{H}_{nn} = \frac{1}{6\pi\eta a}\mathbf{I}$$

$$\mathbf{H}_{nm} = 0 \qquad (n \neq m)$$

II) Oseen tensor:

$$\mathbf{H}_{nn} = \frac{1}{6\pi\eta a}\mathbf{I}$$

$$\mathbf{H}_{nm} = \frac{1}{8\pi\eta r}\left[\mathbf{I} + \frac{\mathbf{rr}}{r^2}\right] \qquad (n \neq m)$$

III) Rotne-Prager-Yamakawa (RPY) tensor:

$$\mathbf{H}_{nn} = \frac{1}{6\pi\eta a}\mathbf{I}$$

$$\mathbf{H}_{nm} = \frac{1}{8\pi\eta r}\left[\mathbf{I} + \frac{\mathbf{rr}}{r^2} + \frac{2}{3}\left(\frac{a}{r}\right)^2\left(\mathbf{I} - \frac{3\mathbf{rr}}{r^2}\right)\right] \qquad (n \neq m)$$

Here,

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{r} = \mathbf{R}_n - \mathbf{R}_m, \quad r = |\mathbf{r}|$$